AD-A130 749 A NOTE ON THE FUNCTIONAL ESTIMATION OF VALUES OF HIDDEN VARIABLES --- AN..(U) ARIZONA STATE UNIV TEMPE GROUP FOR COMPUTER STUDIES OF STRATE.. N V FINDLER ET AL. 1/1

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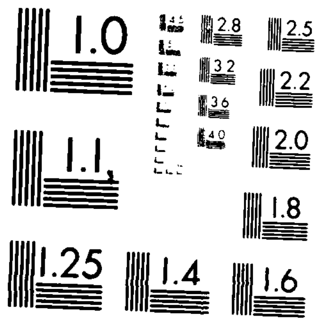

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20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

The paper describes an extension of the authors' work on the Generalized Production Rules System. In its original form, it could estimate at a given point of time or space the value of hidden variables --- variables that can be measured only intermittently or periodically. In contrast, open variables are readily measurable any time. The system establishes stochastic, causal relations, generalized production rules, between known values of hidden variables and certain mathematical properties of the open variables' behavior. These rules are then used to make the point estimates. (CONTINUED)

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ITEM #20, CONTINUED:

The authors have now provided the system with the additional ability to estimate the functional behavior of the hidden variables. The system can serve as a domain-independent module to a knowledge-based expert system in need of such numerical estimates.

AFOSR-TR- 0593

A NOTE ON THE FUNCTIONAL ESTIMATION OF VALUES

OF HIDDEN VARIABLES -- AN EXTENDED MODULE FOR EXPERT SYSTEMS

Nicholas V. Findler and Ron Lo

Group for Computer Studies of Strategies

Computer Science Department

Arizona State University

Tempe, AZ 85287

AFOSR-82-0340

83 07 26 164

ABSTRACT

The paper describes an extension of ~~our~~ the authors' work on the Generalized Production Rules System. In its original form, it could estimate at a given point of time or space the value of hidden variables -- variables that can be measured only intermittently or periodically. In contrast, open variables are readily measurable any time. The system establishes stochastic, causal relations, generalized production rules, between known values of hidden variables and certain mathematical properties of the open variables' behavior. These rules are then used to make the point estimates.

~~We~~ The system have now provided the system with the additional ability to estimate the functional behavior of the hidden variables. The system can serve as a domain-independent module to a knowledge-based expert system in need of such numerical estimates.

INTRODUCTION AND BRIEF SUMMARY OF PRIOR WORK

We have reported on various theoretical and practical aspects of the Generalized Production Rules System (GPRS) in [1-3]. The reader is referred to the companion paper of this one [4], in which a detailed account is given of the background, motivation, methodology and the results of the project. However, a short summary of the underlying ideas of GPRS below will make this article understandable on its own.

Strategic decision-making aims at achieving one's own goals and preventing the achievement of the adversaries' goals over a sequence of confrontations. In order to do so, the decision-maker needs to know the values of relevant variables at various times. Some of these variables, the open variables, are readily measurable at any time. Others, the hidden variables, can be measured only at certain times, intermittently or periodically.

The rules in the knowledge base of GPRS connect causally and stochastically related open and hidden variables. Both the causes and effects can be open or hidden variables. The objective of the system is to provide increasingly sharper estimates of the values of the hidden variables as both the number and the quality of the rules increase.

The prediction or estimation of the hidden variable (HV) value is based on a subset of the mathematical properties of the open variable (OV) distribution. Let us assume that we have a sequence of values of an OV over time or space (called 'lag variable' for reasons shown later). A part of the system, the Morph-Fitting Program [5], constructs a unique mathematical description of the behavior of the OV in question by identifying the patterns prevailing over the domain of the lag variable. The mathematical description consists of an ordered set of parametrized basic patterns, called by us morphs, that fit the OV datapoints optimally. Optimality refers to the requirement that a minimum number of morphs are computed for a prespecified level of statistical significance (that is, there is a tolerated level of "unexplained" variance around the morphs).

FIGURE 1 ABOUT HERE

A morph can be one of the three basic patterns as shown on Figure 1:

.a trend is a monotonic change, a straight line, with three parameters: length, slope and base (starting) value;

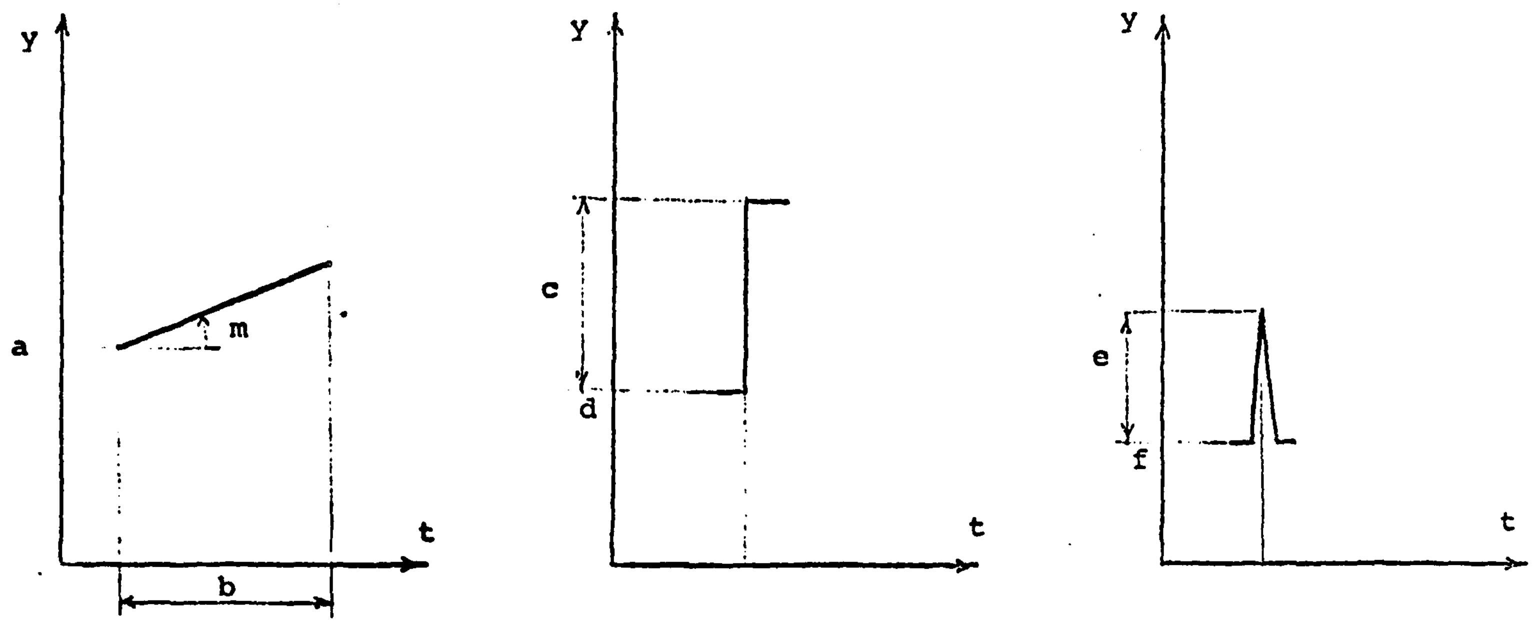
.a step function connects the end point of a trend with the starting point of another if there is a discontinuity between two adjacent trends, and has two parameters: base value and change;

.a sudden change is a momentary jump superimposed onto a trend, with two parameters: base value and peak.

MFP also identifies a fourth basic pattern, the delay function. It covers a segment of the lag variable over which the OV datapoints are too "scattered" to be described mathematically. Its only parameter is its length. Since the delay function represents a lack of information about the OV behavior, it is not used in the predictor part of the generalized production rules.

The *r*-th rule, expressing a stochastic and causal relation, has the following form

$$W_r/M_{ijk}/T_{jm} \dashrightarrow V_m(H_n) \quad : Q_r \tag{1}$$



a: base value,
b: length,
m: slope;

trend

d: base value,
c: change;

step function

f: base value,
e: peak.

sudden change

Figure 1

The left hand, <u>predictor</u> part of the GPR replaces the "condition" of the usual production rule. It consists of three components in the $\underline{r}$-th rule:

(1) The number of similar rules pooled together through a learning process to form the $\underline{r}$-th rule, W_r;

(2) The values of the $\underline{i}$-th combination of the parameters that characterize the $\underline{j}$-th morph describing the $\underline{k}$-th OV, M_{ijk}. Whenever the morph has two parameters (step function or sudden change), the "combination parameter", $\underline{i}$, can be 1, 2 or 3; i.e. one or the other or both parameters of the morph reference the rule. The trends have three parameters, in which case there are seven possible parameter combinations; any single ($\underline{i}$ = 1,...,3) or pair of ($\underline{i}$ = 4,...,6) or all three parameters ($\underline{i}$ = 7) may reference the rule.

(3) The difference in lag variable, T_{jm}, between the start of the $\underline{j}$-th morph (when it is a trend) or the occurrence of the $\underline{j}$-th morph (when it is a step function or a sudden change), on one hand, and the point when the $\underline{n}$-th hidden variable, H_n, assumes its $\underline{m}$-th value, V_m, on the other. (See the right-hand side of (1).) This difference may be positive -- in case OV is the cause and, therefore, precedes the HV, the effect -- or negative in the opposite case.

Finally, the last symbol on the right-hand side, Q_r, stands for the credibility level of the $\underline{r}$-th rule. Its value is between 0 and 1, and depends on two factors:

.how well the morph in question fits the datapoints over its domain, and

.how many and how similar the rules were that have been pooled together to form the rule at hand.

Datapoints representing OV and corresponding lag variable values may be either read in from pre-existing files or typed in by the user. Similarly, occasionally available HV values and their corresponding lag variable values are put on separate files. The user then invokes the MFP component which converts the "raw" data on the sequence of OV values into basic pattern descriptions. He can then direct the system to set up all applicable rules -- tentatively causal relations. However, the interactive system enables one to reduce the probability of a combinatorially explosive situation in which every morph parameter combination of every OV is associated with every value of every HV. The user is asked to specify

.which OV's and HV's are likely to be causally related;

.which is the cause and which is the effect in a given OV-HV combination (the sign of the lag);

.the upper and lower limits of the lag value between a given OV and HV (limits of relevance and physical possibility).

As time proceeds, more data become available and more rules are established. These rules that represent 'real' causal relations will recur but their parameters may vary somewhat due to measurement errors, changes in the environmental conditions, etc. Such rules are similar and should be combined. Also, the credibility level of the pooled rule must be raised in view of the new evidence corroborating the causal association between the OV and HV in question. The parameters of the pooled rule are the weighted average values of those of the contributing rules.

The rules are then ordered according to decreasing values of the credibility level. Those that were established by chance co-occurrence will thus percolate down in the hierarchy and will not be employed.

When an estimate of an HV value is desired at a certain lag variable value, the user has to provide a sequence of values of one or several OV's in its "vicinity", that is within the range of meaningful causal relation between them. The OV values are then submitted to MFP and the system looks in the knowledge base for rules of highest credibility that

.connect the HV sought and the available OV's;

.refer to the same morph type as obtained with the new datapoints;

.involve morph parameter and lag variable values that are "similar enough" to those in the query; i.e., that are within the user-specified range of pooling rules.

The user can ask for a given number, _N_, best estimates. The returned _N_ values are not necessarily those obtained from the top _N_ rules satisfying the above conditions. Namely, the _overall_ quality of an estimate, its _confidence level_, depends on two additional factors and, therefore, the quality ordering of the estimates may differ from the order of the rules used. These factors are

.how well the new morph fits its datapoints,

.how close the parameters of the new morph are to those of the morph matched in the knowledge base.

These criteria have been translated into probabilistic measures and the system computes as many as possible, up to N, ordered estimates of the HV.

Finally, we mention two interesting features of the GPRS. We have made provisions for distributed processing and intelligence. Satellite computers can collect data and establish regional knowledge bases. The user at the central node of a star-like network has the option of merging both data files or knowledge bases if these satisfy certain statistical criteria of compatibilty whose parameters were set by the user with reference to the generation and combination of rules.

As another option, the user can display the English transcription of any single or all the rules of a specified segment of the knowledge base. He can also get the User's Manual on the screen, and is directed and guided by the system continually as to his responses to system questions.

ON FUNCTIONAL ESTIMATION

The program to compute the functional estimate is the sixth and final phase of the GPRS project before we use it for practical applications.

The basic idea can be expressed by the following algorithm:

A1: Using MFP, fit morphs to the new "predictor" dataset of OV's.

A2: Identify in the knowledge base all rules with matching ("similar enough") morph parameters.

A3: Make all point estimates, prescribed by the rules, which are above a user-specified confidence level.

A4: Calculate the range of validity of each point estimate. This is a function of the lag value in the rule used and the user-specified tolerance level in rule-pooling (See Appendix A).

A5: Each continuous concatenation of the ranges of point estimates becomes the range of validity of the functional estimates. (The possible gaps between them render the functional estimates disjoint.)

A6: Use MFP to fit morphs to the point estimates weighted by their confidence level.

A7: Compute the quality of the functional estimates, using the probabilty-theoretic method built in the MFP.

Some additional explanation of the above steps of the algorithm is in order.

E1: This part of the program is unchanged from that used for point estimation. The highly interactive system guides and directs the user in specifying the required parameters.

E2: Again, as with the previous version of GPRS, an optimized search process identifies matching rules above a certain credibility level.

E3: Instead of a user-specified number of ordered best estimates, the system returns all point estimates, over the relevant ranges of the lag variable, which are above a given confidence level.

E4: Each of the point estimates has several ranges of validity associated with it, depending on which rule in the knowledge base was used for estimation.

E5: The program concatenates the overlapping ranges of validity of the point estimates to obtain (possibly disjoint) ranges of validity for the functional estimates.

E6: Each point estimate is duplicated over its range of validity by a number of times that is proportional to the confidence level of the estimate. This multiplicity of points is then used by MFP to fit morphs through them. The simple mechanism of duplicating datapoints makes sure that proper weighting is given to them in the calculation of the morph parameters.

E7: The goodness-of-fit by the morphs is readily available through the MFP.

Finally, the plotting facility provides a visual aid in assessing the value of the functional estimates obtained.

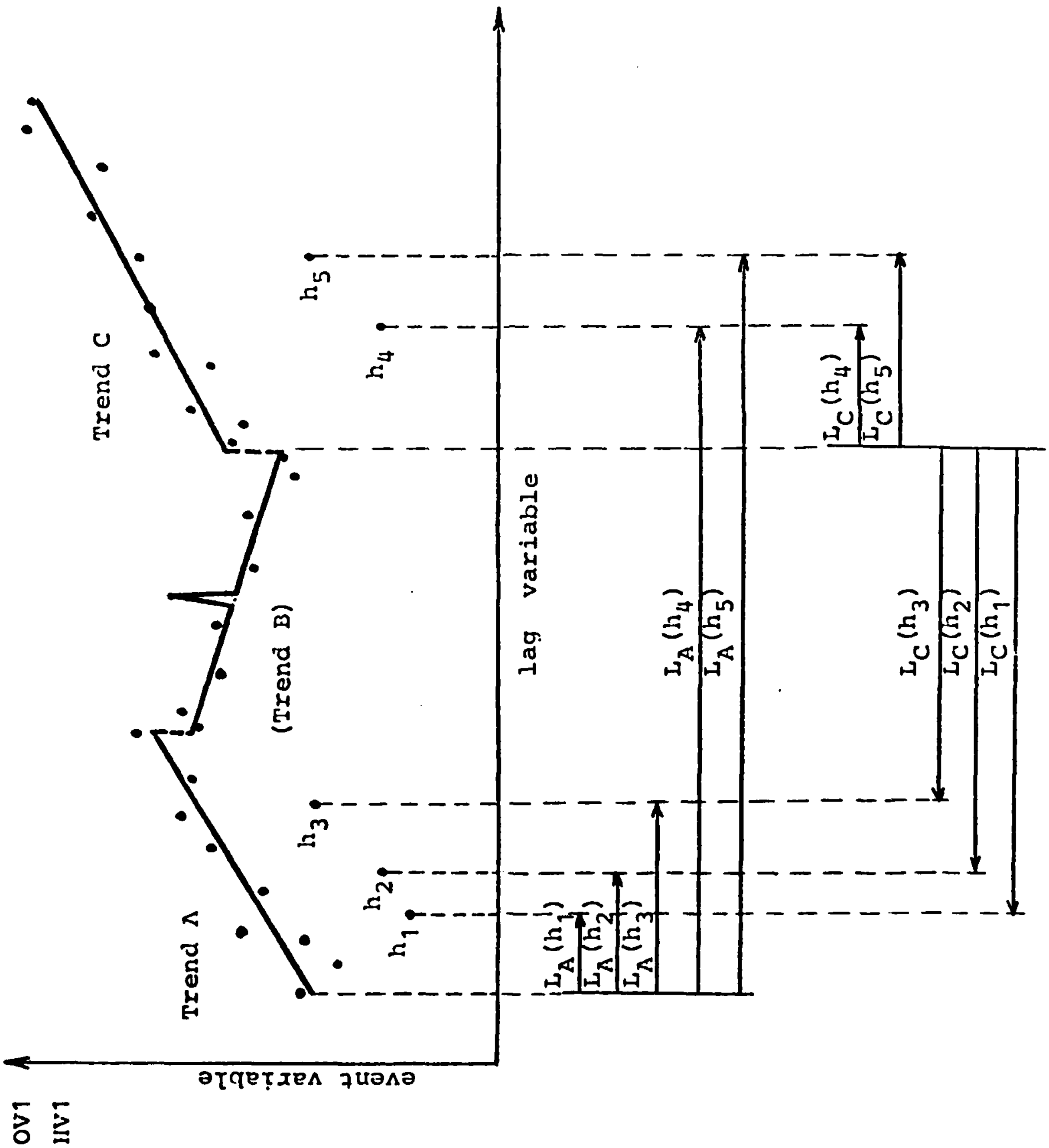
Figure 2-a shows an excerpt from a hypothetical knowledge base. Three trends, two step functions and a sudden change are the morphs which characterize the behavior

of OV1. The parameters of these morphs are associated in the rules with each of the five HV1 values, $\underline{h_1}$ to $\underline{h_5}$. Only the slope of Trends A and C will be matched by the "predictor" data, as described below. Therefore, Figure 2-a shows 10 lag values. $\underline{L_i}(\underline{h_j})$ denotes the (signed) distance between the starting point of Trend $\underline{i}$ and the lag variable value belonging to $\underline{h_j}$, where $\underline{i}$ = $\underline{A}$ or $\underline{C}$ and $\underline{j}$ = 1,...,5.

Figure 2-b contains two trends and a step function describing the "predictor" datapoints of OV1. Only the slope of Trend E matches knowledge base morph parameters, namely the slope of both Trends A and C. The point estimates thus computed are $\underline{h_1'}$, $\underline{h_2'}$, $\underline{h_3'}$ (through rules with Trend C); $\underline{h_1''}$, $\underline{h_2'''}$, $\underline{h_3'''}$, (through rules with Trend A); and $\underline{h_2''}$, $\underline{h_3''}$ (through rules with both Trends A and C). Note that the user did not tell which of OV1 and HV1 is the cause and which is the effect. The respective ranges of validity of each point estimate are also given.

Figure 2-c shows the point estimate values duplicated. Since $\underline{h_2''}$ and $\underline{h_3''}$ are predicted through two rules each, their confidence level and, therefore, multiplicity of duplication are higher than those of the other points. Morphs are then fitted through the duplicated points and the confidence levels of the functional estimates are computed.

FIGURE 2 ABOUT HERE



Trend A
(Trend B)
Trend C
lag variable
event variable
h1
h2
h3
h4
h5
LA(h1)
LA(h2)
LA(h3)
LA(h4)
LA(h5)
LC(h1)
LC(h2)
LC(h3)
LC(h4)
LC(h5)

Figure 2-a

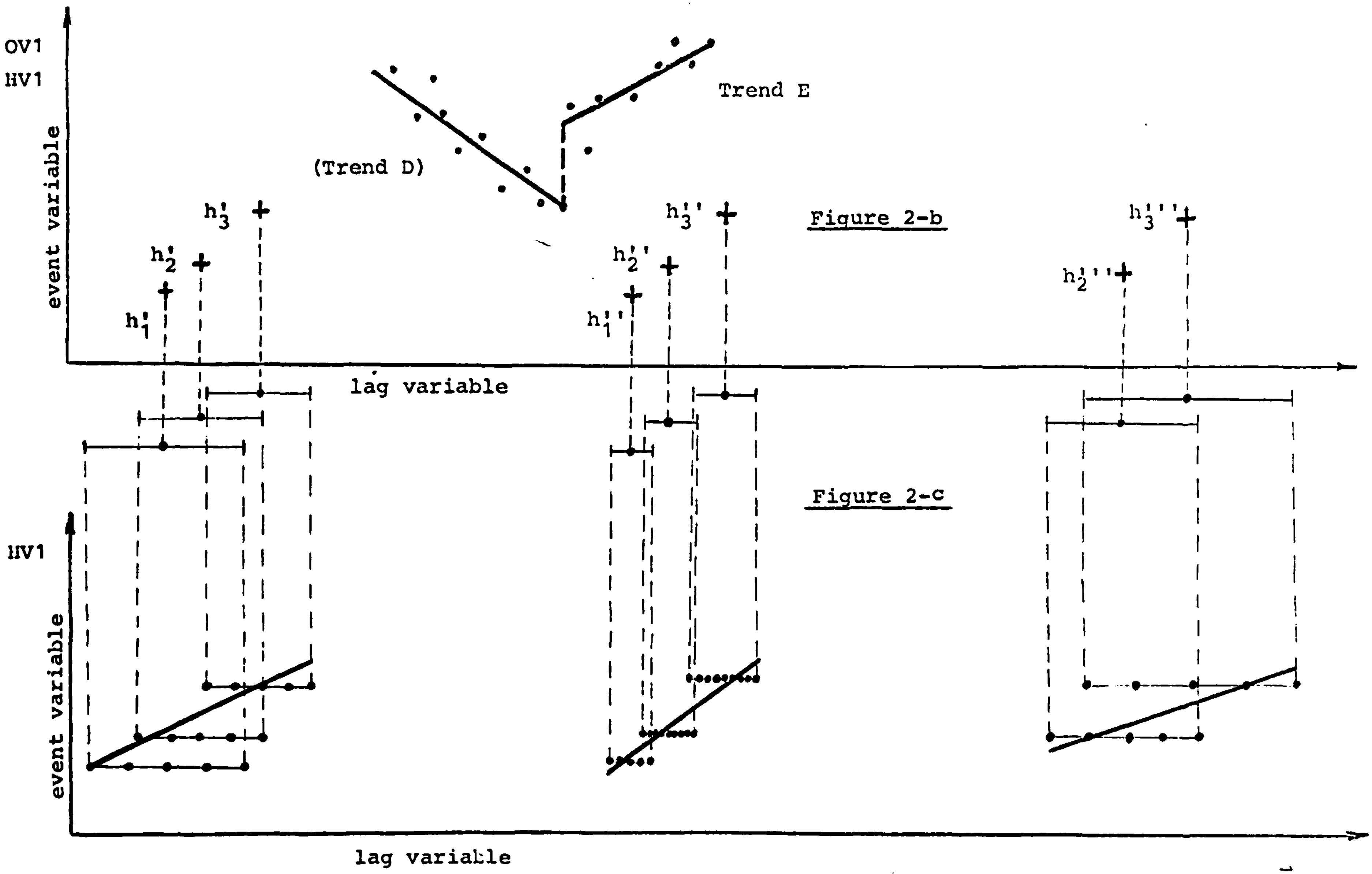


Figure 2-b

Figure 2-c

ACKNOWLEDGEMENTS

The project has been supported by AFOSR Grant 81-0220. The interface between ALISP and FORTRAN IV was written by Don McKay. Ernesto Morgado programmed the Morph-Fitting Program. John E. Brown and Han Yong You contributed to the implementation of GPRS. Nancy Strohmeier collected the data on the macro-economic variables. Michael Belofsky did the word processing for this paper.

REFERENCES

[1] Findler, N. V.: Pattern recognition and generalized production systems in strategy development (Proc. Fifth Internat. Conf. on Pattern Recognition, Vol. I, pp. 140-145, 1980).

[2] Findler, N. V.: A multi-level learning technique using production systems (Cybernetics and Systems, 13, pp. 25-30, 1982).

[3] Findler, N. V.: An expert subsystem based on Generalized Production Rules (Submitted for publication).

[4] Findler, N. V., J. E. Brown, R. Lo, and H. Y. You: A module to estimate numerical values of hidden variables for expert systems (Submitted for publication).

[5] Findler, N. V. and E. Morgado: Morph-fitting -- An effective technique of approximation (Submitted for publication).

APPENDIX A

The following shows how to calculate the range of validity of a point estimate. Let us define the notation first.

H : the HV-end point of the predicted lag;

O : the OV-end point of the predicted lag. This can be either the start of a trend, or the occurrence of a step function or of a sudden change;

T : the relative difference allowed between morph parameters of poolable rules;

L : the lag parameter of the rule in the knowledge base whose associated morph matches the one describing the "predictor" OV data. It can be positive (OV is the cause and HV is effect) or negative (the other way around).

It is then true that

$$(1 - T/2).L \leq H - O \leq (1 + T/2).L \quad \text{for } L \geq 0, \quad \text{(A1)}$$

$$(1 + T/2).L \leq H - O \leq (1 - T/2).L \quad \text{for } L < 0. \quad \text{(A2)}$$

Therefore, the boundary points of the range of validity of a point estimate at H are [(1 - T/2).L + O] and [(1 + T/2).L + O] regardless of the sign of the lag in the rule invoked.

APPENDIX B

An exemplary dialog is given between the user and the GPRS providing point and functional estimates.

(BEGIN)

W E L C O M E   TO

T H E

```
  +++++    +++++++    +++++++
 +++++++   ++++++++   ++++++++
+++   +++  +++   +++  +++   +++
+++   +++  +++   +++  +++   +++
+++        +++   +++  +++   +++
+++        ++++++++   ++++++++
+++ +++++  +++++++    +++++++
+++ +++++  +++        +++ +++
+++   +++  +++        +++  +++
+++   +++  +++        +++   +++
 +++++++   +++        +++   +++
  +++++    +++        +++   +++
```

S Y S T E M

PLEASE TYPE 'C' TO CONTINUE
?C

----- I N D E X -----

(1). INTRODUCTION.

(2). USER'S MANUAL.

(3). LOADING DATA FOR OPEN VARIABLE(S).

(4). LOADING DATA FOR HIDDEN VARIABLE(S).

(5). PREDICTION.

(6). DISPLAYING RULES IN THE KNOWLEDGE BASE(S).

(7). MERGING TWO KNOWLEDGE BASES.

(8). MERGING TWO SOURCE FILES.

(9). ENQUIRING ABOUT THE CONTENTS OF SOURCE FILE(S)

(10). END OF SESSION ...... EXIT.

```
SELECT ONE OF THE ABOVE AND TYPE A NUMBER BETWEEN 1 AND 10
 ?5
IS '5' WHAT YOU WANT TO INPUT (Y/N)
 ?Y
PLEASE TYPE IN THE NAME OF THE FILE CONTAINING THE KNOWLEDGE BASE
 ?KB
IS 'KB' WHAT YOU WANT TO INPUT (Y/N)
 ?Y
WHAT IS THE NAME OF THE OPEN VARIABLE
 ?OV1
IS 'OV1' WHAT YOU WANT TO INPUT (Y/N)
 ?Y
HAVE YOU ALREADY PREPARED A DATA FILE (Y/N)
 ?Y
IS 'Y' WHAT YOU WANT TO INPUT (Y/N)
 ?Y
DO YOU WANT TO ADD MORE DATA TO IT (Y/N)
 ?N
IS 'N' WHAT YOU WANT TO INPUT (Y/N)
 ?Y
WHAT IS THE NAME OF THE INPUT FILE
 ?OVPRED
IS 'OVPRED' WHAT YOU WANT TO INPUT (Y/N)
 ?Y
** THE DATA FILE IS NOW BEING PROCESSED.
THE MFP IS RUN AS A SUBMIT JOB.......

WHAT IS THE NAME OF THE HIDDEN VARIABLE WHICH YOU WANT TO PREDICT
 ?HV1
```

```
IS 'HV1' WHAT YOU WANT TO INPUT (Y/N)
 ?Y
PLEASE SPECIFY THE LOWER BOUND OF THE CONFIDENCE LEVEL FOR
ANY POSSIBLE PREDICTIONS
 ?0.650
IS '0.650' WHAT YOU WANT TO INPUT (Y/N)
 ?Y
PLEASE TYPE IN 'C' TO SEE IF THE O.V. DATASET HAS BEEN PROCESSED
 ?C
MORPHS HAVE ALREADY BEEN FITTED TO THE O.V. DATA, THANKS FOR WAITING.


THE RANGE(S) OF VALIDITY FOR FUNCTIONAL ESTIMATION IS(ARE):
_____________________________________________________________NO. OF RULES
                                                              INVOKED
  1     FROM: 301.8040 LAG UNIT    TO: 302.2510 LAG UNIT.       2
  2     FROM: 302.7319 LAG UNIT    TO: 304.9250 LAG UNIT.       10
  3     FROM: 310.6940 LAG UNIT    TO: 311.0860 LAG UNIT.       1
  4     FROM: 311.6219 LAG UNIT    TO: 312.7690 LAG UNIT.       2


     (1)   POINT ESTIMATION FOR H.V.'HV1'.

     (2)   FUNCTIONAL ESTIMATION FOR H.V.'HV1'.

     (3)   OUTPUT THE RANGE(S) OF VALIDITY FOR FUNCTIONAL ESTIMATION.

     (4)   PROVIDE MORE PREDICTIONS OF ANOTHER HIDDEN VARIABLE.

SELECT ONE OF THE ABOVE AND TYPE A NUMBER BETWEEN 1 AND 4
 ?2
IS '2' WHAT YOU WANT TO INPUT (Y/N)
 ?Y
PLEASE TYPE IN A LIST WHICH CONTAINS THE RANGE(S) OF VALIDITY
THAT YOU WANT TO PREDICT IN, EXAMPLE: (1 3 4)
 ?(2)
IS '(2)' WHAT YOU WANT TO INPUT (Y/N)
 ?Y
ENTER MINSCOP - (MINIMUM NO. OF POINTS IN A TREND)
 ?6
IS '6' WHAT YOU WANT TO INPUT (Y/N)
 ?Y
ENTER RDELTAL - (THRESHOLD FOR R TO ADD POINTS)
 ?0.250
IS '0.250' WHAT YOU WANT TO INPUT (Y/N)
 ?Y
ENTER RDELTAH - (THRESHOLD FOR R TO DROP POINTS)
 ?0.500
IS '0.500' WHAT YOU WANT TO INPUT (Y/N)
 ?Y
ENTER FDELTAL - (THRESHOLD FOR F TO ADD POINTS)
 ?0.400
```

```
IS '0.400' WHAT YOU WANT TO INPUT (Y/N)
 ?Y
ENTER FDELTAH - (THRESHOLD FOR F TO DROP POINTS)
 ?0.050
IS '0.050' WHAT YOU WANT TO INPUT (Y/N)
 ?Y
ENTER YSIZE - (SIZE OF Y-AXIS FOR PLOTTING)
 ?50
IS '50' WHAT YOU WANT TO INPUT (Y/N)
 ?Y
ENTER TRACE - (0-NONE, 1-SELECTIVE, 2-COMPLETE)
 ?1
IS '1' WHAT YOU WANT TO INPUT (Y/N)
 ?Y
ENTER IPLOT - (0-NO PLOT, 1-PLOT)
 ?0
IS '0' WHAT YOU WANT TO INPUT (Y/N)
 ?Y
** A NUMBER OF PREDICTIONS HAVE BEEN COLLECTED AND PROCESSED.
THE MFP IS NOW RUN AS A SUBMIT FILE...

DO YOU WANT TO HAVE MORE PREDICTIONS (Y/N)
 ?N
IS 'N' WHAT YOU WANT TO INPUT (Y/N)
 ?Y
PLEASE TYPE 'C' TO CONTINUE
 ?C

     (1)  POINT ESTIMATION FOR H.V.'HV1'.

     (2)  FUNCTIONAL ESTIMATION FOR H.V.'HV1'.

     (3)  OUTPUT THE RANGE(S) OF VALIDITY FOR FUNCTIONAL ESTIMATION.

     (4)  PROVIDE MORE PREDICTIONS OF ANOTHER HIDDEN VARIABLE.

SELECT ONE OF THE ABOVE AND TYPE A NUMBER BETWEEN 1 AND 4
 ?4
IS '4' WHAT YOU WANT TO INPUT (Y/N)
 ?Y
DO YOU WANT TO MAKE MORE PREDICTIONS BASED ON THIS O.V. DATA SET (Y/N)
 ?N
IS 'N' WHAT YOU WANT TO INPUT (Y/N)
 ?Y
DO YOU WANT TO ADD THIS O.V. DATA SET INTO THE KNOWLEDGE BASE (Y/N)
 ?N
IS 'N' WHAT YOU WANT TO INPUT (Y/N)
 ?Y
DO YOU WANT TO PREDICT OTHER H.V. VALUE(S) BASED ON ANOTHER SET OF
O.V. OBSERVATIONS (Y/N)
 ?N
```

```
IS 'N' WHAT YOU WANT TO INPUT (Y/N)
 ?Y
DO YOU WANT THE PREDICTION(S) BASED ON ANOTHER OPEN VARIABLE (Y/N)
 ?N
IS 'N' WHAT YOU WANT TO INPUT (Y/N)
 ?Y
** END OF PREDICTION(S)
PLEASE TYPE 'C' TO CONTINUE
 ?C
                    ----- I N D E X -----

              (1).  INTRODUCTION.

              (2).  USER'S MANUAL.

              (3).  LOADING DATA FOR OPEN VARIABLE(S).

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              (8).  MERGING TWO SOURCE FILES.

              (9).  ENQUIRING ABOUT THE CONTENTS OF SOURCE FILE(S)

             (10).  END OF SESSION ...... EXIT.

SELECT ONE OF THE ABOVE AND TYPE A NUMBER BETWEEN 1 AND 10
 ?10
IS '10' WHAT YOU WANT TO INPUT (Y/N)
 ?Y
** THE FOLLOWING ARE THE ESTIMATED JOB COSTS:
---------------------------------------------
 CPU TIME      59.008 SECONDS. $     1.639
 MEMORY         0.569 KWRD-HR. $     0.569
 DISK I/O      71.546 KUNITS.  $     1.431
 TLX-PORT       9.920 MINUTES. $     0.231
 --- APPROXIMATE JOB COST ---  $     3.870

BYE.......
```

Legend for Figures

Figure 1 - Three morph types and their parameters.

Figure 2 - (a) Schematic representation of morphs, lag values and hidden variable values appearing in the rules of a hypothetical knowledge base;

(b) Morphs fitting new open variable data and point estimates computed;

(c) The ranges of validity for point and functional estimates, and the functional estimates.

AFOSR TR- 0593

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Group for Computer Studies of Strategies

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Arizona State University

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INTRODUCTION AND BRIEF SUMMARY OF PRIOR WORK

We have reported on various theoretical and practical aspects of the Generalized Production Rules System (GPRS) in [1-3]. The reader is referred to the companion paper of this one [4], in which a detailed account is given of the background, motivation, methodology and the results of the project. However, a short summary of the underlying ideas of GPRS below will make this article understandable on its own.

Strategic decision-making aims at achieving one's own goals and preventing the achievement of the adversaries' goals over a sequence of confrontations. In order to do so, the decision-maker needs to know the values of relevant variables at various times. Some of these variables, the _open variables_, are readily measurable at any time. Others, the _hidden variables_, can be measured only at certain times, intermittently or periodically.

The rules in the knowledge base of GPRS connect causally and stochastically related open and hidden variables. Both the causes and effects can be open or hidden variables. The objective of the system is to provide increasingly sharper estimates of the values of the hidden variables as both the number and the quality of the rules increase.

The prediction or estimation of the hidden variable (HV) value is based on a subset of the mathematical properties of the open variable (OV) distribution. Let us assume that we have a sequence of values of an OV over time or space (called 'lag variable' for reasons shown later). A part of the system, the Morph-Fitting Program [5], constructs a unique mathematical description of the behavior of the OV in question by identifying the patterns prevailing over the domain of the lag variable. The mathematical description consists of an ordered set of parametrized basic patterns, called by us _morphs_, that fit the OV datapoints optimally. Optimality refers to the requirement that a _minimum_ number of morphs are computed for a prespecified level of statistical significance (that is, there is a tolerated level of "unexplained" variance around the morphs).

FIGURE 1 ABOUT HERE

A morph can be one of the three basic patterns as shown on Figure 1:

.a _trend_ is a monotonic change, a straight line, with three parameters: length, slope and base (starting) value;

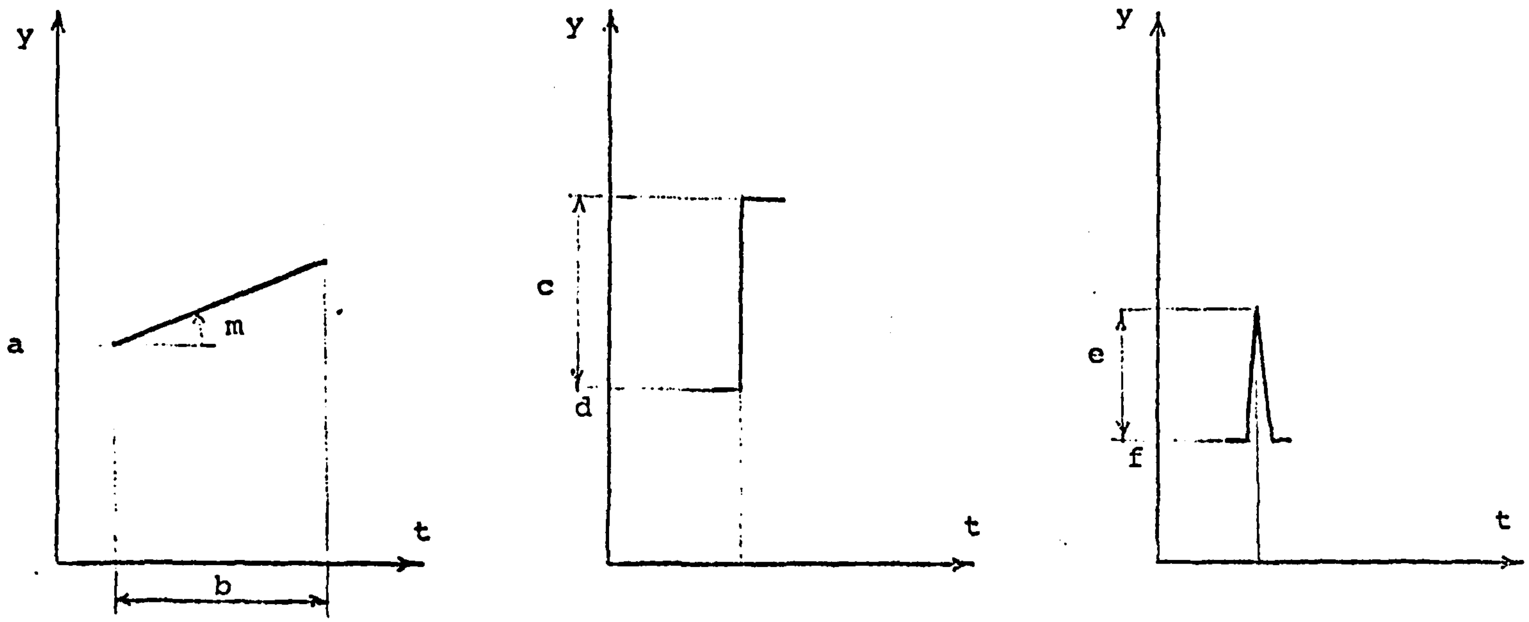
.a _step function_ connects the end point of a trend with the starting point of another if there is a discontinuity between two adjacent trends, and has two parameters: base value and change;

.a _sudden change_ is a momentary jump superimposed onto a trend, with two parameters: base value and peak.

MFP also identifies a fourth basic pattern, the _delay function_. It covers a segment of the lag variable over which the OV datapoints are too "scattered" to be described mathematically. Its only parameter is its length. Since the delay function represents a lack of information about the OV behavior, it is not used in the predictor part of the generalized production rules.

The _r_-th rule, expressing a stochastic and causal relation, has the following form

$$W_r/M_{ijk}/T_{jm} \dashrightarrow V_m(H_n) \quad : Q_r \qquad (1)$$



a: base value,
b: length,
m: slope;

<u>trend</u>

d: base value,
c: change;

<u>step function</u>

f: base value,
e: peak.

<u>sudden change</u>

Figure 1

The left hand, predictor part of the GPR replaces the "condition" of the usual production rule. It consists of three components in the *r*-th rule:

(1) The number of similar rules pooled together through a learning process to form the *r*-th rule, W_r;

(2) The values of the *i*-th combination of the parameters that characterize the *j*-th morph describing the *k*-th OV, M_{ijk}. Whenever the morph has two parameters (step function or sudden change), the "combination parameter", *i*, can be 1, 2 or 3; i.e. one or the other or both parameters of the morph reference the rule. The trends have three parameters, in which case there are seven possible parameter combinations; any single (i = 1,...,3) or pair of (i = 4,...,6) or all three parameters (i = 7) may reference the rule.

(3) The difference in lag variable, T_{jm}, between the start of the *j*-th morph (when it is a trend) or the occurrence of the *j*-th morph (when it is a step function or a sudden change), on one hand, and the point when the *n*-th hidden variable, H_n, assumes its *m*-th value, V_m, on the other. (See the right-hand side of (1).) This difference may be positive -- in case OV is the cause and, therefore, precedes the HV, the effect -- or negative in the opposite case.

Finally, the last symbol on the right-hand side, Q_r, stands for the credibility level of the *r*-th rule. Its value is between 0 and 1, and depends on two factors:

.how well the morph in question fits the datapoints over its domain, and

.how many and how similar the rules were that have been pooled together to form the rule at hand.

Datapoints representing OV and corresponding lag variable values may be either read in from pre-existing files or typed in by the user. Similarly, occasionally available HV values and their corresponding lag variable values are put on separate files. The user then invokes the MFP component which converts the "raw" data on the sequence of OV values into basic pattern descriptions. He can then direct the system to set up all applicable rules -- tentatively causal relations. However, the interactive system enables one to reduce the probability of a combinatorially explosive situation in which every morph parameter combination of every OV is associated with every value of every HV. The user is asked to specify

.which OV's and HV's are likely to be causally related;

.which is the cause and which is the effect in a given OV-HV combination (the sign of the lag);

.the upper and lower limits of the lag value between a given OV and HV (limits of relevance and physical possibility).

As time proceeds, more data become available and more rules are established. These rules that represent 'real' causal relations will recur but their parameters may vary somewhat due to measurement errors, changes in the environmental conditions, etc. Such rules are similar and should be combined. Also, the credibility level of the pooled rule must be raised in view of the new evidence corroborating the causal association between the OV and HV in question. The parameters of the pooled rule are the weighted average values of those of the contributing rules.

The rules are then ordered according to decreasing values of the credibility level. Those that were established by chance co-occurrence will thus percolate down in the hierarchy and will not be employed.

When an estimate of an HV value is desired at a certain lag variable value, the user has to provide a sequence of values of one or several OV's in its "vicinity", that is within the range of meaningful causal relation between them. The OV values are then submitted to MFP and the system looks in the knowledge base for rules of highest credibility that

.connect the HV sought and the available OV's;

.refer to the same morph type as obtained with the new datapoints;

.involve morph parameter and lag variable values that are "similar enough" to those in the query; i.e., that are within the user-specified range of pooling rules.

The user can ask for a given number, N, best estimates. The returned N values are not necessarily those obtained from the top N rules satisfying the above conditions. Namely, the <u>overall</u> quality of an estimate, its <u>confidence level</u>, depends on two additional factors and, therefore, the quality ordering of the estimates may differ from the order of the rules used. These factors are

.how well the new morph fits its datapoints,

.how close the parameters of the new morph are to those of the morph matched in the knowledge base.

These criteria have been translated into probabilistic measures and the system computes as many as possible, up to N, ordered estimates of the HV.

Finally, we mention two interesting features of the GPRS. We have made provisions for distributed processing and intelligence. Satellite computers can collect data and establish regional knowledge bases. The user at the central node of a star-like network has the option of merging both data files or knowledge bases if these satisfy certain statistical criteria of compatibilty whose parameters were set by the user with reference to the generation and combination of rules.

As another option, the user can display the English transcription of any single or all the rules of a specified segment of the knowledge base. He can also get the User's Manual on the screen, and is directed and guided by the system continually as to his responses to system questions.

ON FUNCTIONAL ESTIMATION

The program to compute the functional estimate is the sixth and final phase of the GPRS project before we use it for practical applications.

The basic idea can be expressed by the following algorithm:

A1: Using MFP, fit morphs to the new "predictor" dataset of OV's.

A2: Identify in the knowledge base all rules with matching ("similar enough") morph parameters.

A3: Make all point estimates, prescribed by the rules, which are above a user-specified confidence level.

A4: Calculate the range of validity of each point estimate. This is a function of the lag value in the rule used and the user-specified tolerance level in rule-pooling (See Appendix A).

A5: Each continuous concatenation of the ranges of point estimates becomes the range of validity of the functional estimates. (The possible gaps between them render the functional estimates disjoint.)

A6: Use MFP to fit morphs to the point estimates weighted by their confidence level.

A7: Compute the quality of the functional estimates, using the probabilty-theoretic method built in the MFP.

Some additional explanation of the above steps of the algorithm is in order.

E1: This part of the program is unchanged from that used for point estimation. The highly interactive system guides and directs the user in specifying the required parameters.

E2: Again, as with the previous version of GPRS, an optimized search process identifies matching rules above a certain credibility level.

E3: Instead of a user-specified number of ordered best estimates, the system returns all point estimates, over the relevant ranges of the lag variable, which are above a given confidence level.

E4: Each of the point estimates has several ranges of validity associated with it, depending on which rule in the knowledge base was used for estimation.

E5: The program concatenates the overlapping ranges of validity of the point estimates to obtain (possibly disjoint) ranges of validity for the functional estimates.

E6: Each point estimate is duplicated over its range of validity by a number of times that is proportional to the confidence level of the estimate. This multiplicity of points is then used by MFP to fit morphs through them. The simple mechanism of duplicating datapoints makes sure that proper weighting is given to them in the calculation of the morph parameters.

E7: The goodness-of-fit by the morphs is readily available through the MFP.

Finally, the plotting facility provides a visual aid in assessing the value of the functional estimates obtained.

Figure 2-a shows an excerpt from a hypothetical knowledge base. Three trends, two step functions and a sudden change are the morphs which characterize the behavior

of OV1. The parameters of these morphs are associated in the rules with each of the five HV1 values, $\underline{h_1}$ to $\underline{h_5}$. Only the slope of Trends A and C will be matched by the "predictor" data, as described below. Therefore, Figure 2-a shows 10 lag values. $\underline{L_i}(\underline{h_j})$ denotes the (signed) distance between the starting point of Trend $\underline{i}$ and the lag variable value belonging to $\underline{h_j}$, where $\underline{i}$ = $\underline{A}$ or $\underline{C}$ and $\underline{j}$ = 1,...,5.

Figure 2-b contains two trends and a step function describing the "predictor" datapoints of OV1. Only the slope of Trend E matches knowledge base morph parameters, namely the slope of both Trends A and C. The point estimates thus computed are $\underline{h_1'}$, $\underline{h_2'}$, $\underline{h_3'}$ (through rules with Trend C); $\underline{h_1''}$, $\underline{h_2'''}$, $\underline{h_3'''}$, (through rules with Trend A); and $\underline{h_2''}$, $\underline{h_3''}$ (through rules with both Trends A and C). Note that the user did not tell which of OV1 and HV1 is the cause and which is the effect. The respective ranges of validity of each point estimate are also given.

Figure 2-c shows the point estimate values duplicated. Since $\underline{h_2''}$ and $\underline{h_3''}$ are predicted through two rules each, their confidence level and, therefore, multiplicity of duplication are higher than those of the other points. Morphs are then fitted through the duplicated points and the confidence levels of the functional estimates are computed.

FIGURE 2 ABOUT HERE

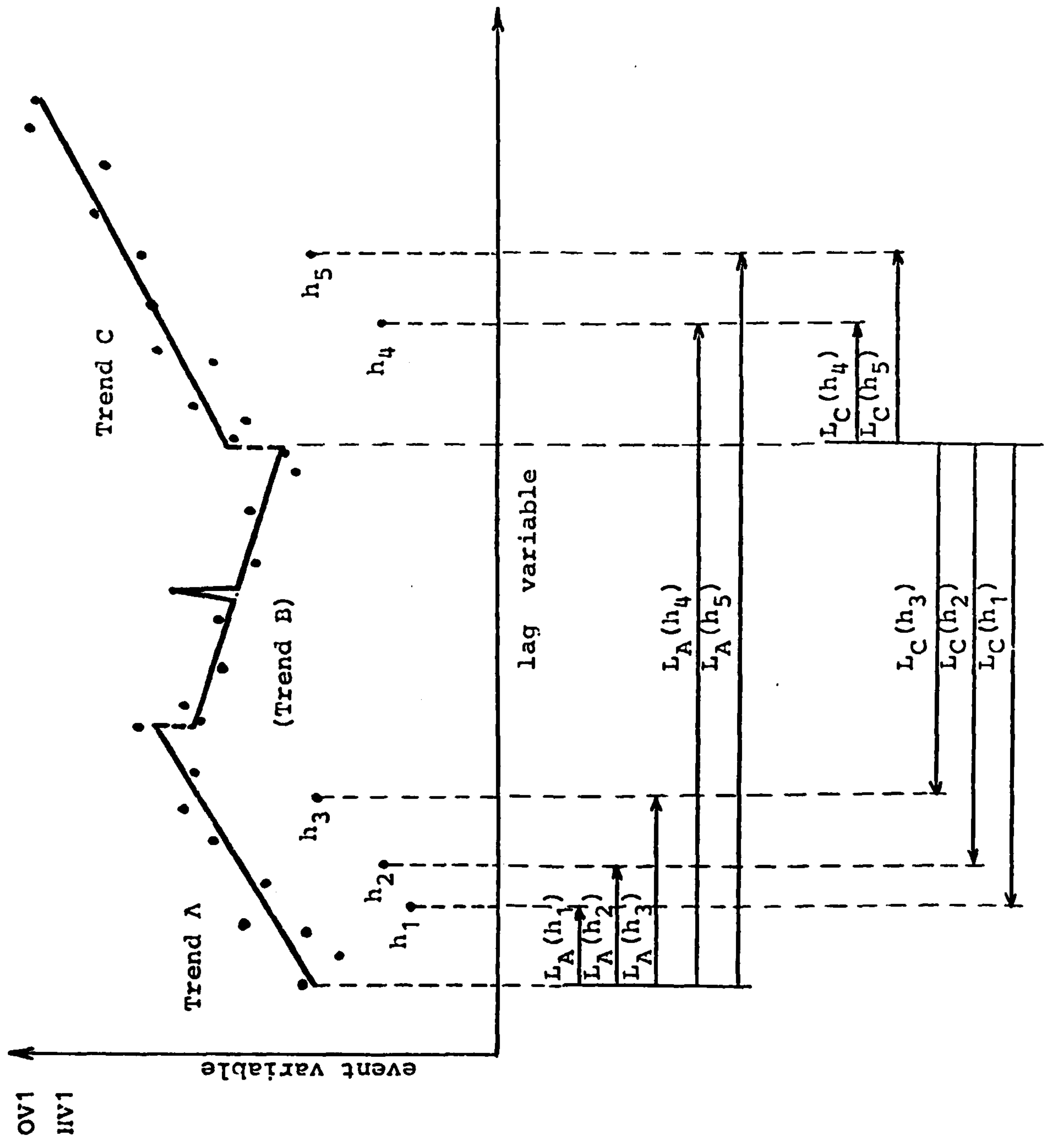

OV1
HV1
event variable
lag variable
Trend A
(Trend B)
Trend C
h1
h2
h3
h4
h5
LA(h1)
LA(h2)
LA(h3)
LA(h4)
LA(h5)
LC(h1)
LC(h2)
LC(h3)
LC(h4)
LC(h5)

Figure 2-a

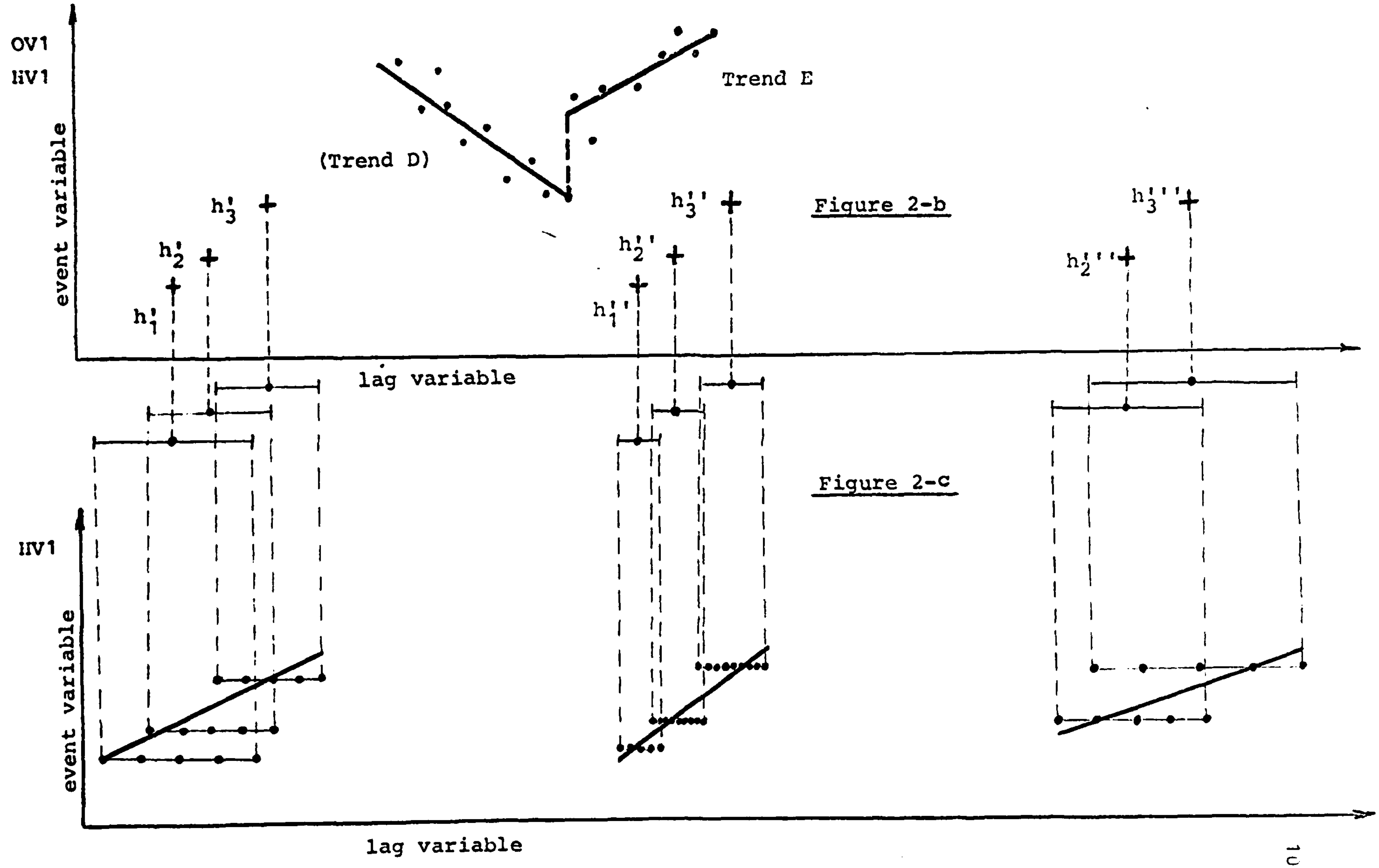


Figure 2-b

Figure 2-c

ACKNOWLEDGEMENTS

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REFERENCES

[1] Findler, N. V.: Pattern recognition and generalized production systems in strategy development (Proc. Fifth Internat. Conf. on Pattern Recognition, Vol. I, pp. 140-145, 1980).

[2] Findler, N. V.: A multi-level learning technique using production systems (Cybernetics and Systems, 13, pp. 25-30, 1982).

[3] Findler, N. V.: An expert subsystem based on Generalized Production Rules (Submitted for publication).

[4] Findler, N. V., J. E. Brown, R. Lo, and H. Y. You: A module to estimate numerical values of hidden variables for expert systems (Submitted for publication).

[5] Findler, N. V. and E. Morgado: Morph-fitting -- An effective technique of approximation (Submitted for publication).

APPENDIX A

The following shows how to calculate the range of validity of a point estimate. Let us define the notation first.

H : the HV-end point of the predicted lag;

O : the OV-end point of the predicted lag. This can be either the start of a trend, or the occurrence of a step function or of a sudden change;

T : the relative difference allowed between morph parameters of poolable rules;

L : the lag parameter of the rule in the knowledge base whose associated morph matches the one describing the "predictor" OV data. It can be positive (OV is the cause and HV is effect) or negative (the other way around).

It is then true that

$$(1 - T/2).L \leq H - O \leq (1 + T/2).L \quad \text{for } L \geq 0, \quad (A1)$$

$$(1 + T/2).L \leq H - O \leq (1 - T/2).L \quad \text{for } L < 0. \quad (A2)$$

Therefore, the boundary points of the range of validity of a point estimate at H are [(1 - T/2).L + O] and [(1 + T/2).L + O] regardless of the sign of the lag in the rule invoked.

APPENDIX B

An exemplary dialog is given between the user and the GPRS providing point and functional estimates.

(BEGIN)

W E L C O M E   TO

T H E

```
  +++++    +++++++    +++++++
 +++++++   ++++++++   ++++++++
+++   +++  +++   +++  +++   +++
+++   +++  +++   +++  +++   +++
+++        +++   +++  +++   +++
+++        ++++++++   ++++++++
+++ +++++  +++++++    +++++++
+++ +++++  +++        +++ +++
+++   +++  +++        +++  +++
+++   +++  +++        +++   +++
 +++++++   +++        +++   +++
  +++++    +++        +++   +++
```

S Y S T E M

PLEASE TYPE 'C' TO CONTINUE
?C

```
----- I N D E X -----
          (1).  INTRODUCTION.
          (2).  USER'S MANUAL.
          (3).  LOADING DATA FOR OPEN VARIABLE(S).
          (4).  LOADING DATA FOR HIDDEN VARIABLE(S).
          (5).  PREDICTION.
          (6).  DISPLAYING RULES IN THE KNOWLEDGE BASE(S).
          (7).  MERGING TWO KNOWLEDGE BASES.
          (8).  MERGING TWO SOURCE FILES.
          (9).  ENQUIRING ABOUT THE CONTENTS OF SOURCE FILE(S)
         (10).  END OF SESSION ...... EXIT.
SELECT ONE OF THE ABOVE AND TYPE A NUMBER BETWEEN 1 AND 10
 ?5
IS '5' WHAT YOU WANT TO INPUT (Y/N)
 ?Y
PLEASE TYPE IN THE NAME OF THE FILE CONTAINING THE KNOWLEDGE BASE
 ?KB
IS 'KB' WHAT YOU WANT TO INPUT (Y/N)
 ?Y
WHAT IS THE NAME OF THE OPEN VARIABLE
 ?OV1
IS 'OV1' WHAT YOU WANT TO INPUT (Y/N)
 ?Y
HAVE YOU ALREADY PREPARED A DATA FILE (Y/N)
 ?Y
IS 'Y' WHAT YOU WANT TO INPUT (Y/N)
 ?Y
DO YOU WANT TO ADD MORE DATA TO IT (Y/N)
 ?N
IS 'N' WHAT YOU WANT TO INPUT (Y/N)
 ?Y
WHAT IS THE NAME OF THE INPUT FILE
 ?OVPRED
IS 'OVPRED' WHAT YOU WANT TO INPUT (Y/N)
 ?Y
** THE DATA FILE IS NOW BEING PROCESSED.
THE MFP IS RUN AS A SUBMIT JOB.......

WHAT IS THE NAME OF THE HIDDEN VARIABLE WHICH YOU WANT TO PREDICT
 ?HV1
```

```
IS 'HV1' WHAT YOU WANT TO INPUT (Y/N)
 ?Y
PLEASE SPECIFY THE LOWER BOUND OF THE CONFIDENCE LEVEL FOR
ANY POSSIBLE PREDICTIONS
 ?0.650
IS '0.650' WHAT YOU WANT TO INPUT (Y/N)
 ?Y
PLEASE TYPE IN 'C' TO SEE IF THE O.V. DATASET HAS BEEN PROCESSED
 ?C
MORPHS HAVE ALREADY BEEN FITTED TO THE O.V. DATA, THANKS FOR WAITING.


THE RANGE(S) OF VALIDITY FOR FUNCTIONAL ESTIMATION IS(ARE):
________________________________________________________________NO. OF RULES
                                                                  INVOKED
  1     FROM: 301.8040 LAG UNIT    TO: 302.2510 LAG UNIT.    2
  2     FROM: 302.7319 LAG UNIT    TO: 304.9250 LAG UNIT.    10
  3     FROM: 310.6940 LAG UNIT    TO: 311.0860 LAG UNIT.    1
  4     FROM: 311.6219 LAG UNIT    TO: 312.7690 LAG UNIT.    2


     (1)  POINT ESTIMATION FOR H.V.'HV1'.

     (2)  FUNCTIONAL ESTIMATION FOR H.V.'HV1'.

     (3)  OUTPUT THE RANGE(S) OF VALIDITY FOR FUNCTIONAL ESTIMATION.

     (4)  PROVIDE MORE PREDICTIONS OF ANOTHER HIDDEN VARIABLE.

SELECT ONE OF THE ABOVE AND TYPE A NUMBER BETWEEN 1 AND 4
 ?2
IS '2' WHAT YOU WANT TO INPUT (Y/N)
 ?Y
PLEASE TYPE IN A LIST WHICH CONTAINS THE RANGE(S) OF VALIDITY
THAT YOU WANT TO PREDICT IN, EXAMPLE: (1 3 4)
 ?(2)
IS '(2)' WHAT YOU WANT TO INPUT (Y/N)
 ?Y
ENTER MINSCOP - (MINIMUM NO. OF POINTS IN A TREND)
 ?6
IS '6' WHAT YOU WANT TO INPUT (Y/N)
 ?Y
ENTER RDELTAL - (THRESHOLD FOR R TO ADD POINTS)
 ?0.250
IS '0.250' WHAT YOU WANT TO INPUT (Y/N)
 ?Y
ENTER RDELTAH - (THRESHOLD FOR R TO DROP POINTS)
 ?0.500
IS '0.500' WHAT YOU WANT TO INPUT (Y/N)
 ?Y
ENTER FDELTAL - (THRESHOLD FOR F TO ADD POINTS)
 ?0.400
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```
IS '0.400' WHAT YOU WANT TO INPUT (Y/N)
 ?Y
ENTER FDELTAH - (THRESHOLD FOR F TO DROP POINTS)
 ?0.050
IS '0.050' WHAT YOU WANT TO INPUT (Y/N)
 ?Y
ENTER YSIZE - (SIZE OF Y-AXIS FOR PLOTTING)
 ?50
IS '50' WHAT YOU WANT TO INPUT (Y/N)
 ?Y
ENTER TRACE - (0-NONE, 1-SELECTIVE, 2-COMPLETE)
 ?1
IS '1' WHAT YOU WANT TO INPUT (Y/N)
 ?Y
ENTER IPLOT - (0-NO PLOT, 1-PLOT)
 ?C
IS '0' WHAT YOU WANT TO INPUT (Y/N)
 ?Y
** A NUMBER OF PREDICTIONS HAVE BEEN COLLECTED AND PROCESSED.
THE MFP IS NOW RUN AS A SUBMIT FILE...

DO YOU WANT TO HAVE MORE PREDICTIONS (Y/N)
 ?N
IS 'N' WHAT YOU WANT TO INPUT (Y/N)
 ?Y
PLEASE TYPE 'C' TO CONTINUE
 ?C

     (1)  POINT ESTIMATION FOR H.V.'HV1'.

     (2)  FUNCTIONAL ESTIMATION FOR H.V.'HV1'.

     (3)  OUTPUT THE RANGE(S) OF VALIDITY FOR FUNCTIONAL ESTIMATION.

     (4)  PROVIDE MORE PREDICTIONS OF ANOTHER HIDDEN VARIABLE.

SELECT ONE OF THE ABOVE AND TYPE A NUMBER BETWEEN 1 AND 4
 ?4
IS '4' WHAT YOU WANT TO INPUT (Y/N)
 ?Y
DO YOU WANT TO MAKE MORE PREDICTIONS BASED ON THIS O.V. DATA SET (Y/N)
 ?N
IS 'N' WHAT YOU WANT TO INPUT (Y/N)
 ?Y
DO YOU WANT TO ADD THIS O.V. DATA SET INTO THE KNOWLEDGE BASE (Y/N)
 ?N
IS 'N' WHAT YOU WANT TO INPUT (Y/N)
 ?Y
DO YOU WANT TO PREDICT OTHER H.V. VALUE(S) BASED ON ANOTHER SET OF
O.V. OBSERVATIONS (Y/N)
 ?N
```

```
  WHAT YOU WANT TO INPUT (Y/N)

  WANT THE PREDICTION(S) BASED ON ANOTHER OPEN VARIABLE (Y/N)

  WHAT YOU WANT TO INPUT (Y/N)

  OF PREDICTION(S)
  TYPE 'C' TO CONTINUE

                    ----- I N D E X -----

              (1).  INTRODUCTION.

              (2).  USER'S MANUAL.

              (3).  LOADING DATA FOR OPEN VARIABLE(S).

              (4).  LOADING DATA FOR HIDDEN VARIABLE(S).

              (5).  PREDICTION.

              (6).  DISPLAYING RULES IN THE KNOWLEDGE BASE(S).

              (7).  MERGING TWO KNOWLEDGE BASES.

              (8).  MERGING TWO SOURCE FILES.

              (9).  ENQUIRING ABOUT THE CONTENTS OF SOURCE FILE(S)

             (10).  END OF SESSION ...... EXIT.

SELECT ONE OF THE ABOVE AND TYPE A NUMBER BETWEEN 1 AND 10
 ?10
IS '10' WHAT YOU WANT TO INPUT (Y/N)
 ?Y
** THE FOLLOWING ARE THE ESTIMATED JOB COSTS:
-------------------------------------------
 CPU TIME      59.008 SECONDS. $     1.639
 MEMORY         0.569 KWRD-HR. $     0.569
 DISK I/O      71.546 KUNITS.  $     1.431
 TLX-PORT       9.920 MINUTES. $     0.231
 --- APPROXIMATE JOB COST ---  $     3.870

BYE.......
```

Legend for Figures

Figure 1 - Three morph types and their parameters.

Figure 2 - (a) Schematic representation of morphs, lag values and hidden variable values appearing in the rules of a hypothetical knowledge base;

(b) Morphs fitting new open variable data and point estimates computed;

(c) The ranges of validity for point and functional estimates, and the functional estimates.

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